Year 10 Autumn 2
Knowledge Organiser

Algebra Basics

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Substitution: Replace the letters with the values.

e.g **Evaluate** (find the **value** of) the expressions, given that:

$$a = 2, \quad b = 3, \quad c = -5$$

1. $4b = 4 \times 2 = \mathbf{8}$
2. $7b - 3c = (7 \times 3) - (3 \times -5) = 21 - ^{-}15 = 21 + 15 = \mathbf{36}$
3. $5b^2 + 1 = 5 \times (3)^2 + 1 = 5 \times 9 + 1 = 45 + 1 = \mathbf{46}$
4. $2c^3 = 2 \times (-5)^3 = 2 \times -125 = \mathbf{-250}$
5. $\frac{3ac}{2b} = \frac{3 \times 2 \times -5}{2 \times 3} = \frac{-30}{6} = \mathbf{-5}$

Building knowledge: Simplifying expressions (multiplying)

- Multiply the numbers and terms separately
- Remember your rules of indices

e.g. $5p \times 3q \times 4p = 60p^2q$

$5 \times 3 \times 4 \times p \times p \times q$

$60 \times p^2q = 60p^2q$

Essential knowledge:

$a + a + a = 3a$

$4 \times d = 4d$

$y \times y \times y = y^3$

$7 \times e \times f = 7ef$

Solving Equations: The balancing method

To work out the value of an unknown in an equation, do the inverse operation to both sides:

$$10x + 3 = 23$$
$$-3 \qquad -3$$
$$10x = 20$$
$$\div 10 \qquad \div 10$$
$$x = 2$$

Collecting Like terms: (Addition and Subtraction)

- Can only simplify like terms.
- Be sure to include the sign before the term

e.g. $2a + 3b - a + 4b = a + 7b$

$2a - a = a \qquad +3b + 4b = +7b$

Exam Questions

For exam questions on some of these topics, scan the QR Codes!

Forming Equations

Changing the subject

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Brackets

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Expanding Single Brackets:

- To expand single brackets, multiply what is outside the bracket by everything inside:
- A number or a letter next to a bracket means multiplied by.
- Remember laws of indices when letters are outside the bracket

E.g. Expand 3(5a - 2)

$$3(5a-2) = (3\times 5a) - (3\times 2)$$

$$= 15a - 6$$

Factorising Single Brackets:

Factorising is the opposite of expanding.

- Find the HCF of both terms.

e.g. 1 $10a + 15 = 5(2a + 3)$

10 & 15 both in the 5 times table

$10a = 5\times 2a$ $\quad 15 = 5\times 3$

e.g. 2 $6x^2 - 21xy = 3x(2x - 7y)$

6 & 21 both in the 3 times table
Both terms have an x in them

$6x^2 = 3x\times 2x$ $\quad 21xy = 3x\times 7y$

Expanding Double Brackets:

To expand double brackets, multiply everything in the first bracket by everything in the second bracket.

Then collect like terms

$$(x+3)(x+5)$$

$$x\times x + x\times 5 + 3\times x + 3\times 5$$

$$x^2 + 5x + 3x + 15$$

$$x^2 + 8x + 15$$

Factorising Double Brackets:

To factorise (put back into brackets), find two numbers th
add to make the middle numb
and multiply to make the last number.

$$x^2 + 16x + 63$$

Add to make this — Multiply to make this

Expanding Double Brackets

Factorising Double Brackets

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Inequalities

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Reading and Writing Inequalities

The list of integers for $-2 < x \leq 1$ is -1, 0, 1.

Check the symbols carefully, if they have the line underneath they include the end value.

Greater than	$>$	Greater than or equal to	$\geq$
Less than	$<$	Less than or equal to	$\leq$
Not equal to	$\neq$		

An open circle means that the value is not included:

$x > 2$ — x is greater than 2

The arrow points in the same direction as the inequality.

A filled in circle means that the value is included:

$x \geq 3$ — x is greater than or equal to 2

If x is between two values, use two circles:

$1 < x \leq 6$

x is greater than 1, but less than or equal to 6.

Solving one step equations/inequalities

To solve any equation or inequality we need to do the inverse of the operation that we see.

$t + 4 = 10$

$-4 \quad -4$

$t = 6$

The inverse of add is subtract and vice versa.

$c - 3 > 6$

$+3 \quad +3$

$c > 9$

$6y < 30$

$\div 6 \quad \div 6$

$y < 5$

The inverse of multiply is divide and vice versa.

$\frac{m}{7} = 4$

$\times 7 \quad \times 7$

$m = 28$

Solving two step equations/inequalities

To solve a two step equation or inequality we need to complete 2 inverse calculations in a specific order.

$6y + 2 = 32$

$-2 \quad -2$

Subtract first because the 2 is separate from the y.

$6y = 30$

$\div 6 \quad \div 6$

Divide because it is the inverse of multiplying.

$y = 5$

$\frac{w-5}{3} \geq 4$

$\times 3 \quad \times 3$

Multiply first because the entire expression is divided by 3.

$w - 5 \geq 12$

$+5 \quad +5$

Add because it is the inverse of subtracting.

$w \geq 17$

KEY POINT:

Inequalities behave like equations when we solve them, but remember, the solution will often be a range of values as opposed to just one unique solution!

Inequalities

Solving Inequalities

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Quadratics

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SOLVING QUADRATICS

Step 1—Try factorising first

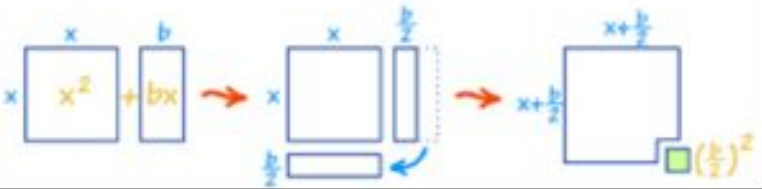
Because it is the easiest method!
Write your equation in the form (if you can)

$$ax^2 + bx + c = 0$$

If you can find two numbers that add up to b and multiply together to give c you can factorise.

Step 2—Complete the Square

If your quadratic equation won't factorise (doesn't make a complete square) you can have a go at completing the square.



Step 3—Quadratic

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Substitute your values into the formula.

For the quadratic formula to work you must arrange your equation in the form

Quadratic = 0

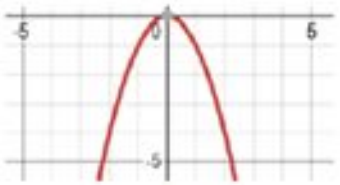
i.e.

$$ax^2 + bx + c = 0$$

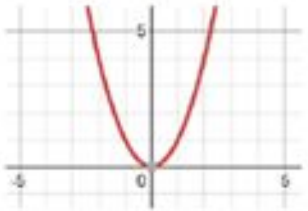
Using this can be faster than completing the square but some questions ask you to use more than one method.

All quadratic graphs have a parabola shape:

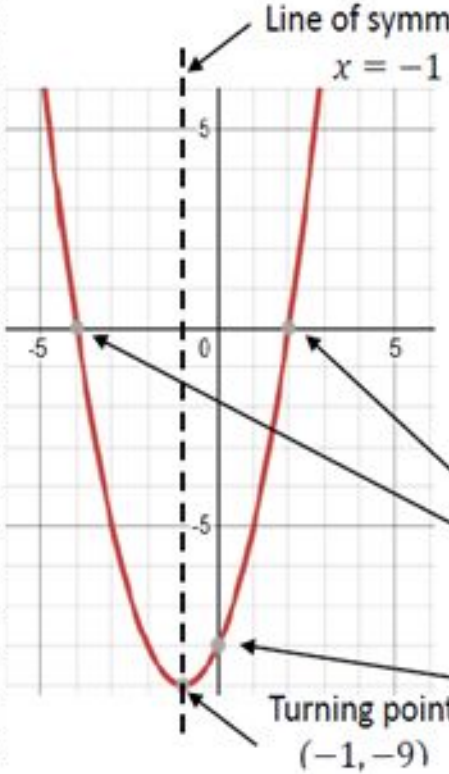
$y = -x^2$



$y = x^2$



The roots of a quadratic graph are where the graph touches the x axis



Examples

$$y = x^2 + 2x - 8$$

A quadratic equation can be solved from its graph.
The roots of the graph tell us the possible solutions for the equation.
There can be 1 root, 2 roots or no roots for a quadratic equation. This is dependant on how many times the graph crosses the x axis.

Roots $x = -4$
$x = 2$

y intercept $= -8$

Quadratic Formula

Solving Quadratics

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Simultaneous Equations

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Simultaneous equations are when **more than one equation** are given, which involve **more than one variable.** The variables have the **same value** in each equation.

Key Words

Simultaneous
Substitution
Elimination
Linear
Quadratic

Two linear equations:

$$3x + 2y = 18$$
$$3x - y = 9 \quad \times 2$$

$$3x + 2y = 18$$
$$6x - 2y = 18 \quad +$$

$$9x = 36$$
$$x = 4$$

Substitute in $x = 4$ into an original equation

$$3x + 2y = 18$$
$$(3 \times 4) + 2y = 18$$
$$12 + 2y = 18$$
$$2y = 6$$
$$y = 3$$

For two linear equations:

Ensure that one of your variables has the same coefficient in each question.

Subtract or add one equation from/to the other to **eliminate** one variable.

Solve your equation which now only contains one variable.

Choose either of the original equations to substitute your new value into.

Solve your equation which now only contains one variable.

You can use both values in the other equation to check your answer.

PRACTICE

Solve this set of simultaneous Equations

1) $3x + 2y = 4$
$4x + 5y = 17$

ANSWERS: 1) $x = -2$ and $y = 5$

Exam Questions