YEAR 9 – CONSTRUCTING IN 2D/3D...

3D Shapes

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Name 2D & 3D shapes
- Recognise Prisms
- Sketch and recognise nets
- Draw plans and elevations
- Find areas of 2D shapes
- Find Surface area for cubes, cuboids, triangular prisms and cylinders
- Find the volume of 3D shapes

Keywords

2D: two dimensions to the shape e.g. length and width

3D: three dimensions to the shape e.g. length, width and height

Vertex: a point where two or more line segments meet

Edge a line on the boundary joining two vertex

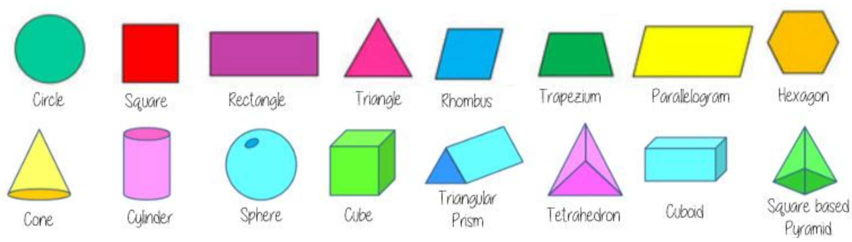
Face: a flat surface on a solid object

Cross-section: a view inside a solid shape made by cutting through it

Plan: a drawing of something when drawn from above (sometimes birds eye view)

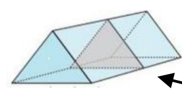
Perspective: a way to give illustration of a 3D shape when drawn on a flat surface.

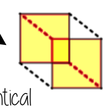
Name 2D & 3D shapes



Recognise prisms

A solid object with two identical ends and flat sides

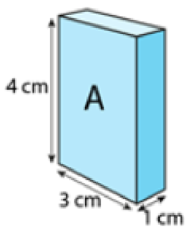


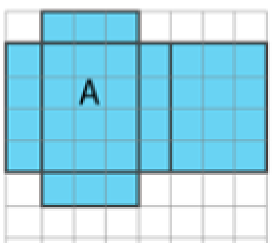


The cross section will also be identical to the end faces.

A cylinder although with very similar properties does not have flat faces so is not categorised as a prism

Nets of cuboids

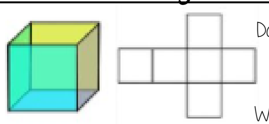


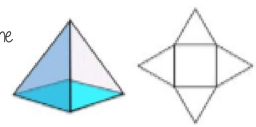


1cm grids help to draw accurately

Visualise the folding of the net. Will it make the cuboid with all sides touching

Sketch and recognise nets

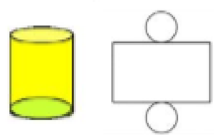


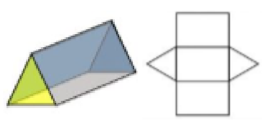


Do they have the same number of faces?

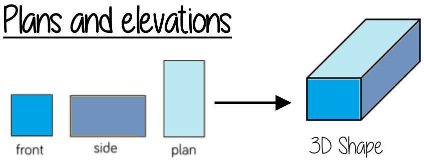
Where do the edges join?

Are the shapes of the faces correct?





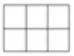
Plans and elevations



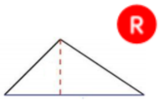
The direction you are considering the shape from determines the front and side views

Area of 2D shapes

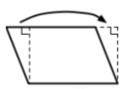
Rectangle
Base x Height



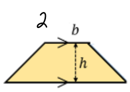
Triangle
½ x Base x Perpendicular height



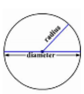
Parallelogram/ Rhombus
Base x Perpendicular height



Area of a trapezium
$\frac{(a+b) \times h}{2}$

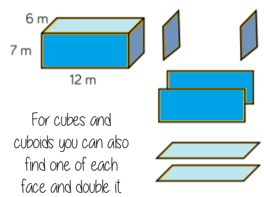


Area of a circle
π x radius²



Surface area

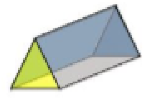
Sketching nets first helps you visualise all the sides that will form the overall surface area



Sides	6 x 7
	6 x 7
Front and back	12 x 7
	12 x 7
Top and Bottom	12 x 6
	12 x 6

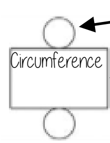
Sum of all sides is surface area

For cubes and cuboids you can also find one of each face and double it



For other shapes – not all the sides are the same, so calculate the individually

Surface area – cylinders



The area of the circle
π x radius²

The width of this face is the same as the circumference
π x diameter x height

2 x π x radius² + π x diameter x height

Volumes

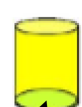
Volume is the 3D space it takes up – also known as capacity if using liquids to fill the space

Counting cubes

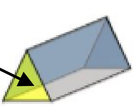
Some 3D shape volumes can be calculated by counting the number of cubes that fit inside the shape

Cubes/ Cuboids = base x width x height

Remember multiplication is commutative



Cross section



Cross section

Prisms and cylinders = area cross section x height

Height can also be described as depth

Areas – square units
Volumes – cube units

Areas and volumes can be left in terms of pi π

YEAR 9 – CONSTRUCTING IN 2D/3D...

Constructions & congruency

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Draw and measure angles
- Construct scale drawings
- Find locus of distance from points, lines, two lines
- Construct perpendiculars from points, lines, angles
- Identify congruence
- Identify congruent triangles

Keywords

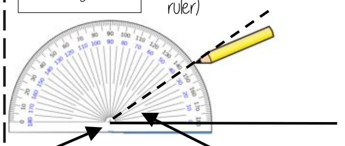
Protractor: piece of equipment used to measure and draw angles
Locus: set of points with a common property
Equidistant: the same distance
Discorectangle: (a stadium) – a rectangle with semi circles at either end
Perpendicular: lines that meet at 90°
Arc: part of a curve
Bisector: a line that divides something into two equal parts
Congruent: the same shape and size

Draw and measure angles

R

Draw a 35° angle

Make a mark at 35° with a pencil
And join to the angle point (use a ruler)



The angle

Make sure the cross is at the end of the line (where you want the angle)

Scale drawings

R

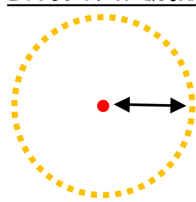
A picture of a car is drawn with a scale of 1:30

For every 1cm on my image is 30cm in real life

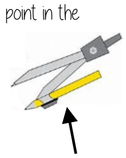
The car image is 10cm

Image : Real life
1cm : 30cm
10cm : 300cm
× 10   × 10

Locus of a distance from a point



All points are equidistant (the same distance) from the fixed point in the middle.



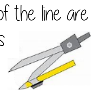
Equipment needed
The radius is the distance from the fixed point

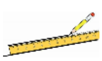
If the point is in the corner it can only make a quarter circle

Locus of a distance from a straight line

All points are equidistant (the same distance) from line

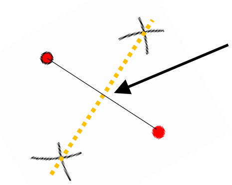
The ends of the line are fixed points





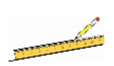
Equipment needed
The line is straight so a ruler is used for the straight lines parallel to your original line

Locus equidistant from two points



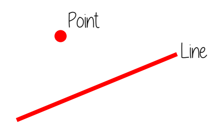
Also a perpendicular bisector
Because if the points are joined, this new line intersects it at a 90°

Keep the compass the same size and draw two arcs from each point

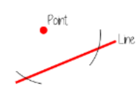


Join the intersections with a ruler.
All points on this line are equidistant from both points

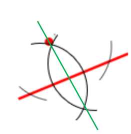
Construct a perpendicular from a point

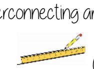


Use a compass and draw an arc that cuts the line. Use the point to place the compass



Keep the compass the same distance and now use your new points to make new interconnecting arcs

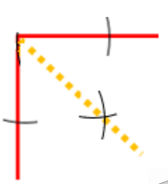




Connecting the arcs makes the bisector

If P is a point on the line the steps are the same

Locus of a distance from two lines



Also an angle bisector
This cuts the angle in half

From the angle vertex draw two arcs that cut the lines forming the angle

Keep the compass the same size and use the new arcs as centres to draw intersecting arcs in the middle

Join the vertex to the intersection

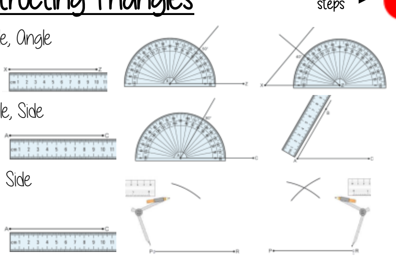
Constructing Triangles

Link to steps → R

Side, Angle, Angle

Side, Angle, Side

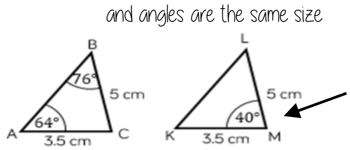
Side, Side, Side



Congruent figures

Congruent figures are identical in size and shape – they can be reflections or rotations of each other

Congruent shapes are identical – all corresponding sides and angles are the same size



$A\hat{C}B = K\hat{M}L$

Because all the angles are the same and AC=KM BC=LM triangles ABC and KLM are **congruent**

Congruent triangles

Side-side-side
All three sides on the triangle are the same size

Angle-side-angle
Two angles and the side connecting them are equal in two triangles

Side-angle-side
Two sides and the angle in-between them are equal in two triangles (it will also mean the third side is the same size on both shapes)

Right angle-hypotenuse-side
The triangles both have a right angle, the hypotenuse and one side are the same

YEAR 9 – REASONING WITH GEOMETRY...

Rotation & Translation

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

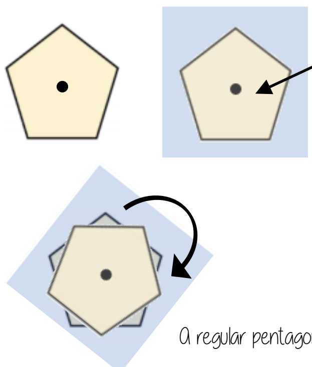
- Identify the order of rotational symmetry
- Rotate a shape about a point on the shape
- Rotate a shape about a point not on a shape
- Translate by a given vector
- Compare rotations and reflections

Keywords

Rotate: a rotation is a circular movement.
Symmetry: when two or more parts are identical after a transformation.
Regular: a regular shape has angles and sides of equal lengths.
Invariant: a point that does not move after a transformation.
Vertex: a point two edges meet.
Horizontal: from side to side
Vertical: from up to down

Rotational Symmetry

Tracing paper helps check rotational symmetry



1. Trace your shape (mark the centre point)
2. Rotate your tracing paper on top of the original through 360°
3. Count the times it fits back into itself

A regular pentagon has rotational symmetry of order 5

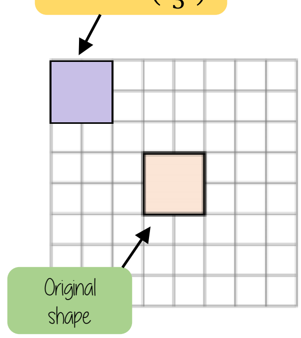
Translation and vector notation

Vector Notation → $\begin{pmatrix}1\\-2\end{pmatrix}$

How far left or right to move
Negative value (left)
Positive value (right)

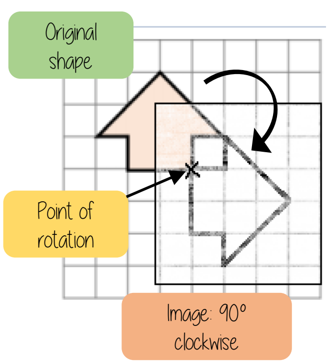
How far up or down to move
Negative value (down)
Positive value (up)

Translation $\begin{pmatrix}-3\\3\end{pmatrix}$



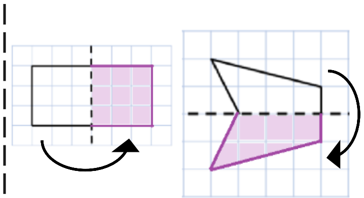
Every vertex has been translated by the same amount

Rotate from a point (in a shape)



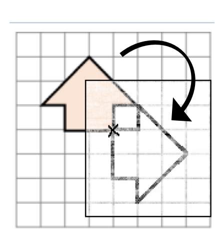
1. Trace the original shape (mark the point of rotation)
2. Keep the point in the same place and turn the tracing paper
3. Draw the new shape

Compare rotations and reflections



Reflections are a mirror image of the original shape.

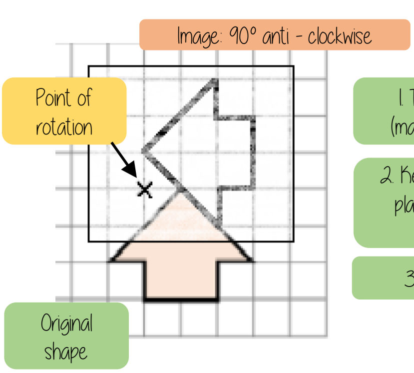
Information needed to perform a reflection:
- Line of reflection (Mirror line)



Rotations are the movement of a shape in a circular motion

Information needed to perform a rotation:
- Point of rotation
- Direction of rotation
- Degrees of rotation

Rotate from a point (outside a shape)



1. Trace the original shape (mark the point of rotation)
2. Keep the point in the same place and turn the tracing paper
3. Draw the new shape

YEAR 9 – REASONING WITH GEOMETRY...

Pythagoras' theorem

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Use square and cube roots
- Identify the hypotenuse
- Calculate the hypotenuse
- Find a missing side in a Right angled triangle
- Use Pythagoras' theorem on axes
- Explore proofs of Pythagoras' theorem

Keywords

Square number: the output of a number multiplied by itself

Square root: a value that can be multiplied by itself to give a square number

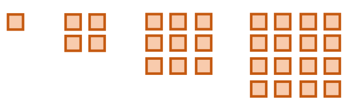
Hypotenuse: the largest side on a right angled triangle. Always opposite the right angle.

Opposite: the side opposite the angle of interest

Adjacent: the side next to the angle of interest

Squares and square roots

R



This can also be written as 6^2

1 × 1	2 × 2	3 × 3	4 × 4	5 × 5	6 × 6	7 × 7	8 × 8	9 × 9	10 × 10
1	4	9	16	25	36	49	64	81	100

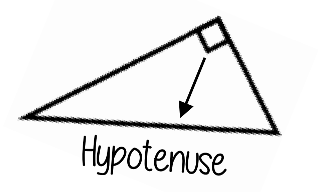
Square numbers

$\sqrt{\ }$ is the square root symbol

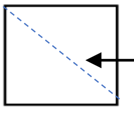
eg $\sqrt{64} = 8$

Because $8 \times 8 = 64$

Identify the hypotenuse

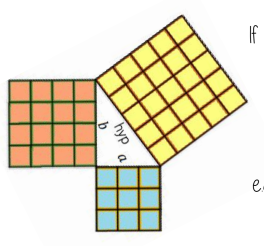


The hypotenuse is always the longest side on a triangle because it is opposite the biggest angle.



Polygons can still have a hypotenuse if it is split up into triangles and opposite a right angle

Determine if a triangle is right-angled



a = 3 b = 4 c = 5

If a triangle is right-angled, the sum of the squares of the shorter sides will equal the square of the hypotenuse.

$$a^2 + b^2 = \text{hypotenuse}^2$$

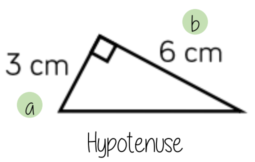
eg $a^2 + b^2 = hypotenuse^2$

$3^2 + 4^2 = 5^2$

$9 + 16 = 25$

Substituting the numbers into the theorem shows that this is a right-angled triangle

Calculate the hypotenuse



Either of the short sides can be labelled a or b

$$a^2 + b^2 = \text{hypotenuse}^2$$

1. Substitute in the values for a and b

$3^2 + 6^2 = \text{hypotenuse}^2$

$9 + 36 = \text{hypotenuse}^2$

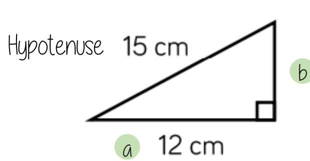
$45 = \text{hypotenuse}^2$

2. To find the hypotenuse square root the sum of the squares of the shorter sides.

$\sqrt{45} = \text{hypotenuse}$

$\mathbf{6.71\,cm} = \text{hypotenuse}$

Calculate missing sides



Either of the short sides can be labelled a or b

$$a^2 + b^2 = \text{hypotenuse}^2$$

$12^2 + b^2 = 15^2$

1 Substitute in the values you are given

$144 + b^2 = 225$

$-144 \qquad -144$

Rearrange the equation by subtracting the shorter square from the hypotenuse squared

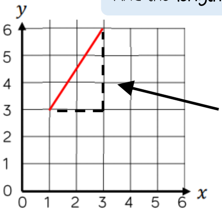
Square root to find the length of the side

$b^2 = 111$

$b = \sqrt{111} = 10.54\ cm$

Pythagoras' theorem on a coordinate axis

Find the **length** of the line segment



The segment can be made into a right-angled triangle by adding the sides on the diagram.

The line segment is the **hypotenuse**

$$a^2 + b^2 = \text{hypotenuse}^2$$

The lengths of a and b are the sides of the triangle.

Be careful to check the scale on the axes