YEAR 7 – REASONING WITH NUMBER

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Prime numbers and Proof

What do I need to be able to do?

By the end of this unit you should be able to:

- Find and use multiples
- Identify factors of numbers and expressions
- Recognise and identify prime numbers
- Recognise square and triangular numbers
- Find common factors including HCF
- Find common multiples including LCM

Keywords

Multiples: found by multiplying any number by positive integers

Factor: integers that multiply together to get another number.

Prime: an integer with only 2 factors.

Conjecture: a statement that might be true (based on reasoning) but is not proven.

Counterexample: a special type of example that disproves a statement.

Expression: a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)

HCF: highest common factor (biggest factor two or more numbers share)

LCM: lowest common multiple (the first time the times table of two or more numbers match)

Multiples

The "times table" of a given number

All the numbers in this lists below are multiples of 3.

3, 6, 9, 12, 15…

$3x, 6x, 9x$ …

This list continues and doesn't end

x could take any value and as the variable is a multiple of 3 the answer will also be a multiple of 3

Non example of a multiple

4.5 is not a multiple of 3 because it is 3 x 1.5

Not an integer

Factors

Arrays can help represent factors

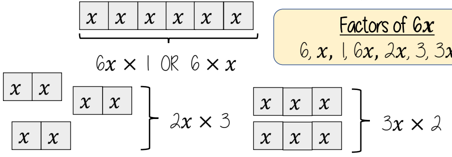
5 x 2 or 2 x 5

Factors of 10
1, 2, 5, 10

10 x 1 or 1 x 10

The number itself is always a factor

Factors and expressions



Factors of $6x$
$6, x, 1, 6x, 2x, 3, 3x, 2$

Prime numbers

- Integer
- Only has 2 factors
- and itself

2

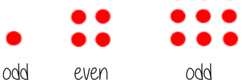
The first prime number
The only even prime number

Learn or how-to quick recall…

2, 3, 5, 7, 11, 13, 17, 19, 23, 29…

Square and triangular numbers

Square numbers



Representations are useful to understand a square number n^2

1, 4, 9, 16, 25, 36, 49, 64 …

Triangular numbers

Representations are useful – an extra counter is added to each new row

Add two consecutive triangular numbers and get a square number

1, 3, 6, 10, 15, 21, 28, 36, 45…

Common factors and HCF

1 is a common factor of all numbers

Common factors are factors two or more numbers share

HCF – Highest common factor

HCF of 18 and 30

18	1, 2, 3, 6, 9, 18
30	1, 2, 3, 5, 6, 10, 15, 30

Common factors
(factors of both numbers)
1, 2, 3, 6

HCF = 6

6 is the biggest factor they share

Common multiples and LCM

Common multiples are multiples two or more numbers share

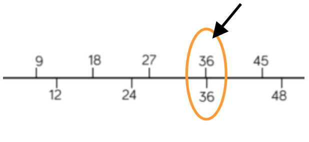
LCM – Lowest common multiple

LCM of 9 and 12

9	9, 18, 27, 36, 45, 54
12	12, 24, 36, 48, 60

LCM = 36

The first time their multiples match



Comparing fractions

$\frac{3}{5}$ and $\frac{7}{10}$

Compare fractions using a LCM denominator

$\frac{6}{10}$ and $\frac{7}{10}$

Conjectures and counterexamples

Conjecture

1, 2, 4…..
The numbers in the sequence are doubling each time.

A pattern that is noticed for many cases

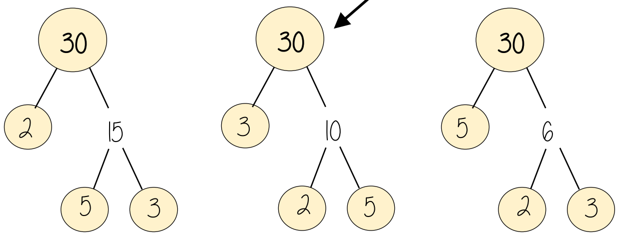
Counterexamples

This sequence isn't doubling it is adding 2 each time

Only **one** counterexample is needed to disprove a conjecture

Product of prime factors

Multiplication part-whole models



All three prime factor trees represent the same decomposition

Multiplication is commutative

30 = 2 x 3 x 5

Multiplication of prime factors

Using prime factors for predictions

eg. 60	30 x 2	2 x 3 x 5 **x 2**
150	30 x 5	2 x 3 x 5 **x 5**

YEAR 8 - ALGEBRAIC TECHNIQUES...

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Indices

What do I need to be able to do?

By the end of this unit you should be able to:

- Add/ Subtract expressions with indices
- Multiply expressions with indices
- Divide expressions with indices
- Know the addition law for indices
- Know the subtraction law for indices

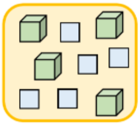
Keywords

Base: The number that gets multiplied by a power
Power: The exponent – or the number that tells you how many times to use the number in multiplication
Exponent: The power – or the number that tells you how many times to use the number in multiplication
Indices: The power or the exponent.
Coefficient: The number used to multiply a variable
Simplify: To reduce a power to its lowest term
Product: Multiply

Addition/ Subtraction with indices

Coefficient   Power

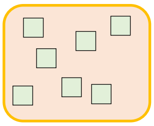
$$\underbrace{\underbrace{5x^2}_{\text{Term}} + \underbrace{4x^4}_{\text{Term}}}_{\text{Expression}} \longrightarrow$$



Each square represents x^2 and each cube represents x^4

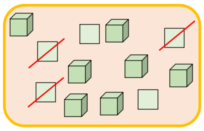
Only similar terms can be simplified
If they have different powers, they are unlike terms

$5x^2 + 2x^2 \longrightarrow$



$\longrightarrow 7x^2$

$5x^2 + 6x^4 - 3x^2 + x^4 \longrightarrow$



$\longrightarrow 2x^2 + 7x^4$

Divide expressions with indices

$$\frac{24}{36} \longrightarrow \frac{\cancel{2} \times \cancel{2} \times 2 \times \cancel{3}}{\cancel{2} \times \cancel{3} \times 2 \times \cancel{3}} \longrightarrow \frac{2}{3}$$

$$\frac{5a^3b^2}{15ab^6} \longrightarrow \frac{\cancel{5} \times \cancel{a} \times a \times a \times \cancel{b} \times \cancel{b}}{\underbrace{3 \times \cancel{5} \times \cancel{a} \times \cancel{b} \times \cancel{b} \times b \times b \times b \times b}} \longrightarrow \frac{a^2}{3b^4}$$

Cross cancelling factors shows cancels the expression

$$\frac{23a^7y^2}{5db^6}$$

This expression cannot be divided (cancelled down) because there are no common factors or similar terms

Multiply expressions with indices

$4b \times 3a$
$\equiv 4 \times b \times 3 \times a$
$\equiv 4 \times 3 \times b \times a$
$\equiv 12ab$

$5t \times 9t$
$\equiv 5 \times t \times 9 \times t$
$\equiv 5 \times 9 \times t \times t$
$\equiv 45t^2$

$2b^4 \times 3b^2$
$\equiv 2 \times b \times b \times b \times b \times 3 \times b \times b$
$\equiv 2 \times 3 \times b \times b \times b \times b \times b \times b$
$\equiv 6b^6$

There are often misconceptions with this calculation but break down the powers

Addition/ Subtraction laws for indices

$$3^5 \times 3^2 = \underbrace{(3 \times 3 \times 3 \times 3 \times 3) \times (3 \times 3)} \longrightarrow 3^7$$

The base number is all the same so the terms can be simplified

Addition law for indices

$$a^m \times a^n = a^{m+n}$$

$$3^5 \div 3^2 \longrightarrow 3^3$$

$$\frac{3 \times 3 \times 3 \times \cancel{3} \times \cancel{3}}{\cancel{3} \times \cancel{3}} \longrightarrow \frac{3^3}{3^0} \longrightarrow \frac{3^3}{1}$$

Subtraction law for indices

$$a^m \div a^n = a^{m-n}$$

YEAR 8 - DEVELOPING NUMBER...

Standard Form

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What do I need to be able to do?

By the end of this unit you should be able to:

- Write numbers in standard form and as ordinary numbers
- Order numbers in standard form
- Add/ Subtract with standard from
- Multiply/ Divide with standard form
- Use a calculator with standard form

Keywords

Standard (index) Form: A system of writing very big or very small numbers

Commutative: an operation is commutative if changing the order does not change the result.

Base: The number that gets multiplied by a power

Power: The exponent – or the number that tells you how many times to use the number in multiplication

Exponent: The power – or the number that tells you how many times to use the number in multiplication

Indices: The power or the exponent.

Negative: A value below zero.

Positive powers of 10

1 billion – 1 000 000 000

$10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10^9$

Addition rule for indices $10^a \times 10^b = 10^{a+b}$

Subtraction rule for indices $10^a \div 10^b = 10^{a-b}$

Standard form with numbers > 1

Any number between 1 and less than 10 → $A \times 10^n$ ← Any integer

Example

3.2×10^4

$= 3.2 \times 10 \times 10 \times 10 \times 10$

$= 32000$

Non-example

0.8×10^4

$5.3 \times 10^{0.7}$

Negative powers of 10

0.001

$1 \times \frac{1}{1000}$

1×10^{-3}

10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
10^1	10^0	10^{-1}	10^{-2}	10^{-3}
0	0	0	0	1

Any value to the power 0 always = 1

Negative powers do not indicate negative solutions

Numbers between 0 and 1

0.054

$= 5.4 \times 10^{-2}$

1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
10^0	10^{-1}	10^{-2}	10^{-3}
0	0	5	4

A negative power does not mean a negative answer – it means a number closer to 0

Order numbers in standard form

10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}	10^{-4}

6.4×10^{-2}	2.4×10^2	3.3×10^0	1.3×10^{-1}
0.064	240	1	0.13

Look at the power first will the number be = > or < than 1

Use a place value grid to compare the numbers for ordering

Mental calculations

$6.4 \times 10^2 \times 1000$ Not in Standard Form

$= 6.4 \times 10^2 \times 10^3$ Use addition for indices rule

$= \underline{6.4 \times 10^5}$

$(2 \times 10^3) \div 4$ Divide the values

$= (2 \div 4) \times 10^3$

$= \underline{0.5 \times 10^3}$

$8 \times 10^5 \times 3$

$= \underbrace{24}_{} \times 10^5$ Not in Standard Form

$= 2.4 \times 10^1 \times 10^5$ Use addition for indices rule

$= \underline{2.4 \times 10^6}$

Remember the layout for standard form

Any number between 1 and less than 10 → $A \times 10^n$ ← Any integer

Addition and Subtraction

Tip: Convert into ordinary numbers first and back to standard from at the end

$6 \times 10^5 + 8 \times 10^5$

Method 1

$= 600000 + 800000$

$= 1400000$

$= \underline{1.4 \times 10^5}$

More robust method
Less room for misconceptions
Easier to do calculations with negative indices
Can use for different powers

Method 2

$= (6 + 8) \times 10^5$

$= 14 \times 10^5$ ← This is not the final answer

$= 1.4 \times 10^1 \times 10^5$

$= \underline{1.4 \times 10^5}$

Only works if the powers are the same

Multiplication and division

$\frac{1.5 \times 10^5}{0.3 \times 10^3}$ ← Division questions can look like this

$(1.5 \times 10^5) \div (0.3 \times 10^3)$

$1.5 \div 0.3 \times 10^5 \div 10^3$

$= \underline{5 \times 10^2}$

For multiplication and division you can look at the values for A and the powers of 10 as two separate calculations

Revisit addition and subtraction laws for indices – they are needed for the calculations

Addition law for indices

$a^m \times a^n = a^{m+n}$

Subtraction law for indices

$a^m \div a^n = a^{m-n}$

Using a calculator

$1.4 \times 10^5 \times 3.9 \times 10^3$

Use a calculator to work out this question to a suitable degree of accuracy.

Input 1.4 and press x10ˣ Then press 5 (for the power)
Press ✕
Input 3.9 and press x10ˣ Then press 3 (for the power)
Press =

This gives you the solution

Click calculator for video tutorial

To put into standard form and a suitable degree of accuracy

Press SHIFT SETUP and then press 7 for sci mode.

Choose a degree of accuracy so in most cases press 2

Answer: 5.5×10^8