YEAR 8 - ALGEBRAIC TECHNIQUES...

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Indices

What do I need to be able to do?

By the end of this unit you should be able to:

- Add/ Subtract expressions with indices
- Multiply expressions with indices
- Divide expressions with indices
- Know the addition law for indices
- Know the subtraction law for indices

Keywords

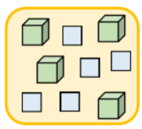
Base: The number that gets multiplied by a power
Power: The exponent — or the number that tells you how many times to use the number in multiplication
Exponent: The power — or the number that tells you how many times to use the number in multiplication
Indices: The power or the exponent.
Coefficient: The number used to multiply a variable
Simplify: To reduce a power to its lowest term
Product: Multiply

Addition/ Subtraction with indices

Coefficient → $5x^2 + 4x^4$ ← Power

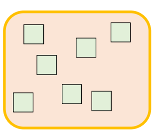
Term, Term

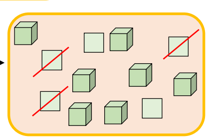
Expression



Each square represents x^2 and each cube represents x^4

Only similar terms can be simplified
If they have different powers, they are unlike terms

$5x^2 + 2x^2$ →  → $7x^2$

$5x^2 + 6x^4 - 3x^2 + x^4$ →  → $2x^2 + 7x^4$

Multiply expressions with indices

$4b \times 3a$
$\equiv 4 \times b \times 3 \times a$
$\equiv 4 \times 3 \times b \times a$
$\equiv 12ab$

$5t \times 9t$
$\equiv 5 \times t \times 9 \times t$
$\equiv 5 \times 9 \times t \times t$
$\equiv 45t^2$

$2b^4 \times 3b^2$
$\equiv 2 \times b \times b \times b \times b \times 3 \times b \times b$
$\equiv 2 \times 3 \times b \times b \times b \times b \times b \times b$
$\equiv 6b^6$

There are often misconceptions with this calculation but break down the powers

Divide expressions with indices

$\frac{24}{36}$ → $\frac{2 \times 2 \times 2 \times 3}{2 \times 3 \times 2 \times 3}$ → $\frac{2}{3}$

$\frac{5a^3b^2}{15ab^6}$ → $\frac{5 \times a \times a \times a \times b \times b}{3 \times 5 \times a \times b \times b \times b \times b \times b \times b}$ → $\frac{a^2}{3b^4}$

Cross cancelling factors shows cancels the expression

$\frac{23a^7y^2}{5db^6}$

This expression cannot be divided (cancelled down) because there are no common factors or similar terms

Addition/ Subtraction laws for indices

$3^5 \times 3^2$ → 3^7
$= (3 \times 3 \times 3 \times 3 \times 3) \times (3 \times 3)$

The base number is all the same so the terms can be simplified

Addition law for indices

$$a^m \times a^n = a^{m+n}$$

$3^5 \div 3^2$ → 3^3

$\frac{3 \times 3 \times 3 \times 3 \times 3}{3 \times 3}$ → $\frac{3^3}{3^0}$ → $\frac{3^3}{1}$

Subtraction law for indices

$$a^m \div a^n = a^{m-n}$$

YEAR 8 - DEVELOPING NUMBER...

Standard Form

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What do I need to be able to do?

By the end of this unit you should be able to:

- Write numbers in standard form and as ordinary numbers
- Order numbers in standard form
- Add/ Subtract with standard from
- Multiply/ Divide with standard form
- Use a calculator with standard form

Keywords

Standard (index) Form: A system of writing very big or very small numbers

Commutative: an operation is commutative if changing the order does not change the result.

Base: The number that gets multiplied by a power

Power: The exponent – or the number that tells you how many times to use the number in multiplication

Exponent: The power – or the number that tells you how many times to use the number in multiplication

Indices: The power or the exponent.

Negative: A value below zero.

Positive powers of 10

1 billion – 1 000 000 000

$10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10^9$

Addition rule for indices $10^a \times 10^b = 10^{a+b}$

Subtraction rule for indices $10^a \div 10^b = 10^{a-b}$

Standard form with numbers > 1

$A \times 10^n$

Any number between 1 and less than 10 → A

Any integer → n

Example

3.2×10^4

$= 3.2 \times 10 \times 10 \times 10 \times 10$

$= 32000$

Non-example

0.8×10^4

$5.3 \times 10^{0.7}$

Negative powers of 10

	10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
0.001					
$1 \times \frac{1}{1000}$	10^1	10^0	10^{-1}	10^{-2}	10^{-3}
1×10^{-3}	0	0	0	0	1

Any value to the power 0 always = 1

Negative powers do not indicate negative solutions

Numbers between 0 and 1

0.054

$= 5.4 \times 10^{-2}$

1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
10^0	10^{-1}	10^{-2}	10^{-3}
0	0	5	4

A negative power does not mean a negative answer – it means a number closer to 0

Order numbers in standard form

10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}	10^{-4}

6.4×10^{-2}	2.4×10^2	3.3×10^0	1.3×10^{-1}
0.064	240	1	0.13

Look at the power first will the number be = > or < than 1

Use a place value grid to compare the numbers for ordering

Mental calculations

$6.4 \times 10^2 \times 1000$ Not in Standard Form

$= 6.4 \times 10^2 \times 10^3$ Use addition for indices rule

$= 6.4 \times 10^5$

$8 \times 10^5 \times 3$

$= 24 \times 10^5$ Not in Standard Form

$= 2.4 \times 10^1 \times 10^5$ Use addition for indices rule

$= 2.4 \times 10^6$

$(2 \times 10^3) \div 4$ Divide the values

$= (2 \div 4) \times 10^3$

$= 0.5 \times 10^3$

Remember the layout for standard form

$A \times 10^n$

Any number between 1 and less than 10 → A

Any integer → n

Addition and Subtraction

Tip: Convert into ordinary numbers first and back to standard from at the end

$6 \times 10^5 + 8 \times 10^5$

Method 1

$= 600000 + 800000$

$= 1400000$

$= 1.4 \times 10^6$

More robust method
Less room for misconceptions
Easier to do calculations with negative indices
Can use for different powers

Method 2

$= (6 + 8) \times 10^5$

$= 14 \times 10^5$ ← This is not the final answer

$= 1.4 \times 10^1 \times 10^5$

$= 1.4 \times 10^6$

Only works if the powers are the same

Multiplication and division

Division questions can look like this

$\frac{1.5 \times 10^5}{0.3 \times 10^3}$

$(1.5 \times 10^5) \div (0.3 \times 10^3)$

$(1.5 \div 0.3) \times 10^5 \div 10^3$

$= 5 \times 10^2$

For multiplication and division you can look at the values for A and the powers of 10 as two separate calculations

Revisit addition and subtraction laws for indices – they are needed for the calculations

Addition law for indices: $a^m \times a^n = a^{m+n}$

Subtraction law for indices: $a^m \div a^n = a^{m-n}$

Using a calculator

$1.4 \times 10^5 \times 3.9 \times 10^3$

Use a calculator to work out this question to a suitable degree of accuracy

Input 1.4 and press [x10^x] Then press 5 (for the power)

Press [x]

Input 3.9 and press [x10^x] Then press 3 (for the power)

Press [=]

This gives you the solution

Click calculator for video tutorial

To put into standard form and a suitable degree of accuracy

Press [SHIFT] [SETUP] and then press 7 for sci mode

Choose a degree of accuracy so in most cases press 2

Answer: 5.5×10^8

YEAR 9 – REASONING WITH ALGEBRA...

Straight Line Graphs

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What do I need to be able to do?

By the end of this unit you should be able to:

- Compare gradients
- Compare intercepts
- Understand and use y= mx + c
- Find the equation of a line from a graph
- Interpret gradient and intercepts of real-life graphs

Keywords

Gradient: the steepness of a line

Intercept: where two lines cross. The y-intercept: where the line meets the y-axis.

Parallel: two lines that never meet with the same gradient.

Co-ordinate: a set of values that show an exact position on a graph.

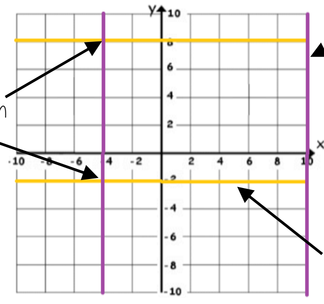
Linear: linear graphs (straight line) – linear common difference by addition/ subtraction

Asymptote: a straight line that a graph will never meet.

Reciprocal: a pair of numbers that multiply together to give 1.

Perpendicular: two lines that meet at a right angle.

Lines parallel to the axes (R)



Intersection points

All the points on this line have a x coordinate of 10

'a' can be ANY positive or negative value including 0

Lines parallel to the **y axis** take the form **x = a** and are **vertical**

Lines parallel to the **x axis** take the form **y = a** and are **horizontal**

All the points on this line have a y coordinate of -2

eg (3, -2) (7, -2) (-2, -2) all lay on this line because the y coordinate is -2

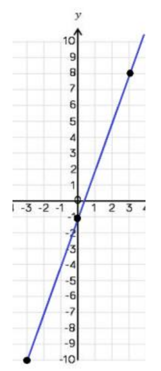
Plotting y = mx + c graphs (R)

$y = 3x - 1$ → 3 x the x coordinate then – 1

x	−3	0	3
y	-10	-1	8

Draw a table to display this information

This represents a coordinate pair (-3, -10)



You only need two points to form a straight line

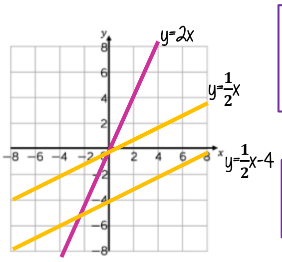
Plotting more points helps you decide if your calculations are correct (if they do make a straight line)

Remember to join the points to make a line

Compare Gradients

$y = mx + c$

The **coefficient** of x (the number in front of x) tells us the gradient of the line



The **greater** the gradient – the **steeper** the line

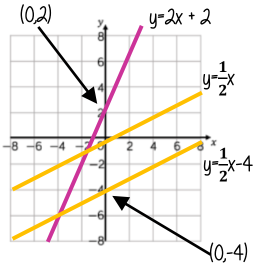
Parallel lines have the **same** gradient

Positive gradients

Negative gradients

Compare Intercepts

$y = mx + c$ ← The value of **c** is the point at which the line crosses the y-axis. **Y intercept**



The coordinate of a y intercept will always be (0,c)

Lines with the **same** y-intercept cross in the **same** place

y = mx + c

The **coefficient** of x (the number in front of x) tells us the gradient of the line

$y = mx + c$ ← The value of **c** is the point at which the line crosses the y-axis. **Y intercept**

y and x are **coordinates**

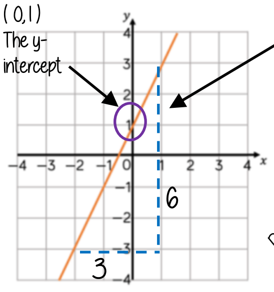
The equation of a line can be rearranged. Eg:

y = c + mx

c = y – mx

Identify which coefficient you are identifying or comparing

Find the equation from a graph



The Gradient $\frac{6}{3} = 2$

$y = 2x + 1$

The direction of the line indicates a positive gradient

Positive gradients

Negative gradients

Real life graphs

A plumber charges a £25 callout fee, and then £12.50 for every hour. Complete the table of values to show the cost of hiring the plumber.

Time (h)	0	1	2	3	8
Cost (£)	£25				£125

The y-intercept shows the minimum charge. The gradient represents the price per mile

In real life graphs like this values will always be positive because they measure distances or objects which cannot be negative.

Direct Proportion graphs To represent direct proportion the graph must start at the origin.

When you have 0 pens this has 0 cost. The gradient shows the price per pen

A box of pens costs £2.30
Complete the table of values to show the cost of buying boxes of pens.

Boxes	0	1	2	3	8
Cost (£)		£2.30			

YEAR 9 – REASONING WITH ALGEBRA…

Forming and Solving Equations

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Solve inequalities with negative numbers
- Solve equations with unknowns on both sides
- Solve inequalities with unknowns on both sides
- Substitute into formulae and equations
- Rearrange formulae

Keywords

Inequality: an inequality compares who values showing if one is greater than, less than or equal to another

Variable: a quantity that may change within the context of the problem

Rearrange: Change the order

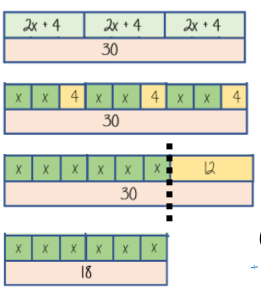
Inverse operation: the operation that reverses the action

Substitute: replace a variable with a numerical value

Solve: find a numerical value that satisfies an equation

Solve equations with brackets

R



$3(2x + 4) = 30$

Expand the brackets

$6x + 12 = 30$

$-12 \quad -12$

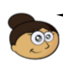
$6x = 18$

$\div 6 \quad \div 6$

$\underline{x = 3}$

Form and solve inequalities

R



Two more than treble my number is greater than 11

Find the possible range of values

$3x + 2 > 11$

Solve

x ← −3 ← −2 ← 11

$x > 3$

Inequalities with negatives

Method 1 Make x positive first

$2 - 3x > 17$

$+3x \quad +3x$

$2 > 17 + 3x$

$-17 \quad -17$

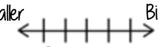
$-15 > 3x$

$\div 3 \quad \div 3$

$\underline{-5 > x}$

x is true for any value smaller than -5

CHECK IT!
$2 - 3(-6) = 20$
TRUE/ CORRECT



Method 2 Keep the negative x

$2 - 3x > 17$

$-2 \quad -2$

$-3x > 15$

$\div -3 \quad \div -3$

$\underline{x > -5}$

x is true for any value bigger than -5

This cannot be true…

$\underline{x < -5}$

When you multiply or divide x by a negative you need to reverse the inequality

Equations with unknown on both sides

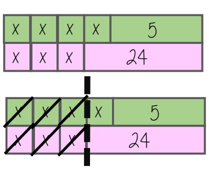
$4x + 5 = 3x + 24$

$-3x \quad -3x$

$x + 5 = 24$

$-5 \quad -5$

$\underline{x = 19}$



Inequalities with unknown on both sides

Solving inequalities has the same method as equations

$5(x + 4) < 3(x + 2)$

$5x + 20 < 3x + 6$

$2x + 20 < 6$

$2x < -14$

$\underline{x < -7}$

Check it!

$5(-8 + 4) < 3(-8 + 2)$

$5(-4) < 3(-6)$

$-20 < -18$

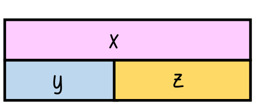
✓ -20 IS smaller than -18

Formulae and Equations

Substitute in values

Formulae – all expressed in symbols → **Equations** – include numbers and can be solved

Rearranging Formulae (one step)



$x = y + z$

Rearrange to make y the subject.

$\underline{y = x - z}$

y → +z → x

y ← −z ← x

Using inverse operations or fact families will guide you through rearranging formulae

Rearranging can also be checked by substitution.

Language of rearranging…

Make XXX the subject

Change the subject

Rearrange

Rearranging Formulae (two step)

In an equation (find x)

$4x - 3 = 9$

$+3 \quad +3$

$4x = 12$

$\div 4 \quad \div 4$

$\underline{x = 3}$

In a formula (make x the subject)

$xy - s = a$

$+s \quad +s$

$xy = a + s$

$\div y \quad \div y$

$x = \dfrac{a + s}{y}$

The steps are the same for solving and rearranging

Rearranging is often needed when using $y = mx + c$

e.g. Find the gradient of the line $2y - 4x = 9$

Make y the subject first $y = \dfrac{4x + 9}{2}$ Gradient $= \dfrac{4}{2} = 2$