YEAR 8 - PROPORTIONAL REASONING...

Ratio and Scale

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Simplify any given ratio
- Share an amount in a given ratio
- Solve ratio problems given a part

Solutions should be modelled, explained and solved.

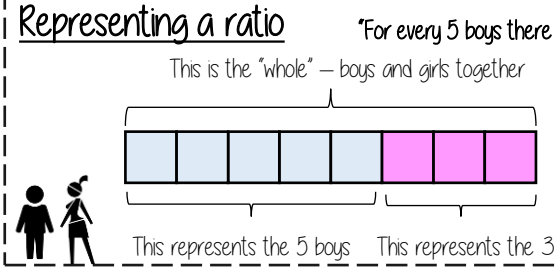
Keywords

Ratio: a statement of how two numbers compare
Equal Parts:: all parts in the same proportion, or a whole shared equally
Proportion: a statement that links two ratios
Order: to place a number in a determined sequence
Part: a section of a whole
Equivalent: of equal value
Factors: integers that multiply together to get the original value
Scale: the comparison of something drawn to its actual size.

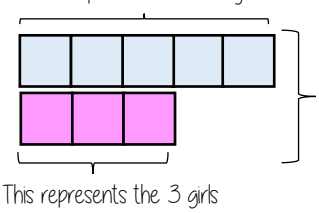
Representing a ratio

"For every 5 boys there are 3 girls"

5:3



Double Number Line



Order is Important

"For every dog there are 2 cats"

Dogs: Cats

1:2

The ratio has to be written in the same order as the information is given.
e.g. 2:1 would represent 2 dogs for every 1 cat ✗

Simplifying a ratio

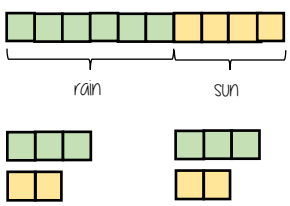
Cancel down the ratio to its lowest form

"For every 6 days of rain there are 4 days of sun"

6:4

÷ by 2

3:2



Find the biggest common factor that goes into all parts of the ratio

For **6** and **4** the biggest factor (number that multiples into them is **2**)

"For every 3 days of rain there are 2 days of sun" – when this happens twice the ratio becomes 6:4

Ratio 1:n (or n:1)

This is asking you to cancel down until the part indicated represents 1

Show the ratio 4:20 in the ratio of 1:n

The question states that **this part has to be** 1 unit. Therefore **Divide by 4**

4 : 20

1 : 5

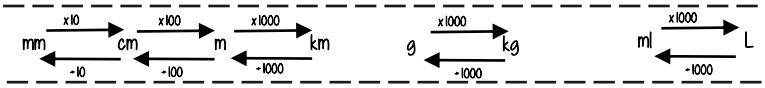
This side has to be divided by 4 too – to keep in **proportion**

H the n part does not have to be an integer for this type of question

Units are important:

When using a ratio – all parts should be in the same units

Useful Conversions



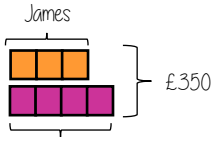
Sharing a whole into a given ratio

James and Lucy share £350 in the ratio 3:4
Work out how much each person earns

Model the Question

James: Lucy

3 : 4



Find the value of one part

Whole: £350
7 parts to share between
(3 James, 4 Lucy)

£350 ÷ 7 = £50

□ = one part
= £50

Put back into the question

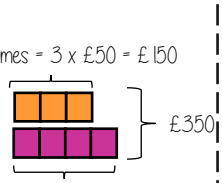
James: Lucy

(x 50) 3 : 4 (x 50)

£150:£200

James = 3 x £50 = £150

Lucy = 4 x £50 = £200



Finding a value given 1:n (or n:1)

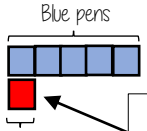
Inside a box are blue and red pens in the ratio 5:1
If there are 10 red pens how many blue pens are there?

Model the Question

Blue : Red

5 : 1

□ = one part
= 10 pens



One unit
= 10 pens

Put back into the question

Blue : Red

(x 10) 5 : 1 (x 10)

50: 10

Blue pens = 5 x 10 = 50 pens

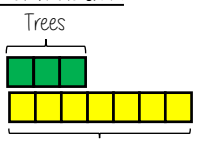
Red pens = 1 x 10 = 10 pens

There are 50 Blue Pens

Ratio as a fraction

Trees: Flowers

3 : 7



Ratio

There are **3** parts for trees

Fraction of trees

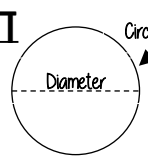
Number of parts of in group / Total number of parts

3/10

Fraction

Tree parts **3** + Flower parts **7** = 10

Pi π



The ratio of a circles circumference to its diameter

YEAR 8 - PROPORTIONAL REASONING...

Multiplicative Change

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What do I need to be able to do?

By the end of this unit you should be able to:

- Solve problems and explain direct proportion
- Use conversion graphs to make statements, comparisons and form conclusions.
- Understand and use scale factors for length

Keywords

Proportion: a statement that links two ratios
Variable: a part that the value can be changed
Axes: horizontal and vertical lines that a graph is plotted around
Approximation: an estimate for a value
Scale Factor: the multiple that increases/ decreases a shape in size
Currency: the system of money used in a particular country
Conversion: the process of changing one variable to another
Scale: the comparison of something drawn to its actual size.

Direct Proportion

As one variable changes the other changes at the same rate

This is a multiplicative change

4 cans of pop = £2.40

4 cans of pop = £2.40 → 2 cans of pop = £1.20 (x 0.5)

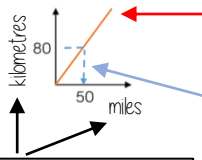
4 cans of pop = £2.40 → 12 cans of pop = £7.20 (x 3)

This multiplier is the same in the same way that this would be for ratio

Sometimes this is easiest if you work out how much one unit is worth first
e.g. 1 can of pop = £0.60

Conversion Graphs

Compare two variables



This is always a straight line because as one variable increases so does the other at the same rate

To make conversions between units you need to find the point to compare – then find the associated point by using your graph.
Using a ruler helps for accuracy
Showing your conversion lines help as a "check" for solutions

Labelling of both axes is vital

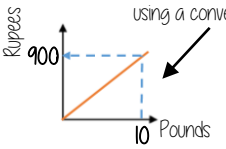
Conversion between currencies

£1 = 90 Rupees ← Currency is directly proportional

For every £1 I have 90 Rupees

£1 = 90 Rupees → £10 = 900 Rupees (x 10)

Currency can be converted using a conversion graph



Convert 630 Rupees into Pounds

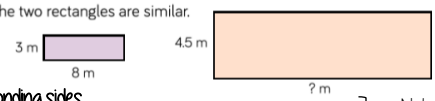
£1 = 90 Rupees → £7 = 630 Rupees (x 7)

630 ÷ 90 = 7

Ratio between similar shapes

Angles in similar shapes do not change
e.g. if a triangle gets bigger the angles can not go above 180°

The two rectangles are similar.



Corresponding sides

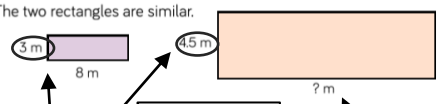
3m : 4.5m → 1m : 1.5 m (÷ 3)

8m : 12m → 1m : 1.5 m (÷ 8)

Note
Simplify to the same ratio

Understand Scale Factor

The two rectangles are similar.



3 x 1.5 = 4.5

This is a multiplicative change

Missing length
8 x 1.5 = 12m

Use **corresponding** sides to calculate a scale factor

Scale factor can also be calculated by:

Bigger corresponding side
Smaller corresponding side

Small corresponding side → Big corresponding side: x SF

Big corresponding side → Small corresponding side: ÷ SF

Draw and interpret scale diagrams

A picture of a car is drawn with a scale of 1:30

For every 1cm on my image is 30cm in real life

The car image is 10cm

Image	Real life
1cm	30cm
10cm	300cm

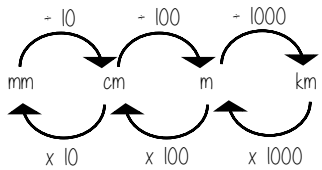
(x 10 / x 10)

The car in real life is 210cm

Image	Real life
1cm	30cm
7cm	210 cm

(x 7 / x 7)

Interpret maps with scale factors



1 cm : 250 m

Ratios need to be in the same units

1 cm : 250m

1 cm : 25000cm

250 x 100 = 25000

For every 1cm on my map is 25000cm in real life.

YEAR 8 - REPRESENTATIONS...

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Working in the Cartesian plane

What do I need to be able to do?

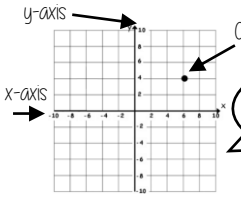
By the end of this unit you should be able to:

- Label and identify lines parallel to the axes
- Recognise and use basic straight lines
- Identify positive and negative gradients
- Link linear graphs to sequences
- Plot y = mx + c graphs

Keywords

Quadrant: four quarters of the coordinate plane.
Coordinate: a set of values that show an exact position.
Horizontal: a straight line from left to right (parallel to the x axis)
Vertical: a straight line from top to bottom (parallel to the y axis)
Origin: (0,0) on a graph. The point the two axes cross
Parallel: Lines that never meet
Gradient: The steepness of a line
Intercept: Where lines cross

Coordinates in four quadrants



Coordinate (x, y) (6, 4)

From the origin this coordinate is 6 places along the positive x axis and 4 places up the positive y axis

(x, y)

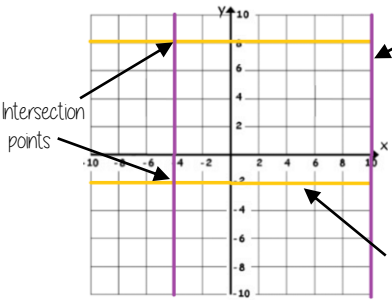
Always the position on the x axis first

Always the position on the y axis second

(0, a) Will be always be a point on the y axis (a can be any number)

(a, 0) Will be always be a point on the x axis (a can be any number)

Lines parallel to the axes



All the points on this line have a x coordinate of 10

'a' can be ANY positive or negative value including 0

Lines parallel to the **y axis** take the form **x = a** and are **vertical**

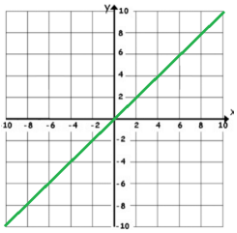
Lines parallel to the **x axis** take the form **y = a** and are **horizontal**

All the points on this line have a y coordinate of -2

e.g. (3, -2) (7, -2) (-2, -2) all lay on this line because the y coordinate is -2

Recognise and use the line y=x

This means the x and the y coordinate have the same value

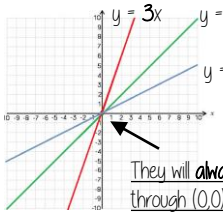


Examples of coordinates on this line: (0, 0) (-3, -3) (8, 8)

The axes **scale is important** – if the scale is the same y = x will be a straight line at 45°

Recognise and use the lines y=kx

The value of **k** changes the steepness of the line



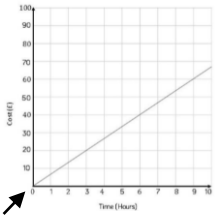
Note: y =x is the same as y= 1x

The bigger the value of k the **steeper** the line will be

They will **always** go through (0,0)

The closer to 0 the value of k the closer the line will be to the x axis.

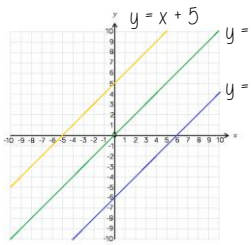
Direct Proportion using y=kx



The line must be straight to be directly proportional – variables increase at the same rate **k**

Direct proportion graphs always start at (0,0) as they are describing relationships between two variables

Lines in the form y = x + a



All the lines are **parallel** because the gradients are the same

y = x + a

This is the line y=x when the y and x coordinate are the same

This shows the translation of that line

e.g y = x + 5 Is the line y=x moved 5 places up the graph

5 has been added to each of the x coordinates

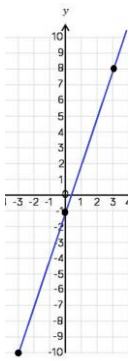
Plotting y = mx + c graphs

$y = 3x - 1$ → 3 x the x coordinate then – 1

x	-3	0	3
y	-10	-1	8

Draw a table to display this information

This represents a coordinate pair (-3, -10)

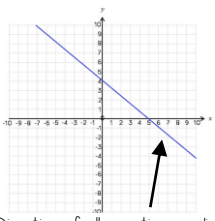


You only need two points to form a straight line

Plotting more points helps you decide if your calculations are correct (if they do make a straight line)

Remember to join the points to make a line

Lines with negative gradients



Any straight-line graph with a negative x value has a negative gradient

E.g y = -2x
y = -x y + x = 12

Direction of all negative gradients