Autumn (Teacher A): Graphs

Skills

- Plot and interpret graphs
- Interpret the gradient of a straight line graph as a rate of change
- Use the form y = mx + c to identify parallel {and perpendicular} lines
- Find the equation of a line given two points, or one point and a given gradient
- Find approximate solutions to two simultaneous equations in two variables (linear/linear or {linear/quadratic}) using a graph
- Sketch, interpret and plot quadratic, simple cubic, reciprocal {and exponential graphs for y = k^x, k >0}
- Find approximate solutions using a graph
- Identify roots and intercepts of quadratic functions
- Plot and solve problems with circle graphs
- Plot and interpret graphs of non-standard functions in real contexts, to find approximate solutions to problems such as simple kinematic problems involving distance, speed and acceleration
- {Apply the concepts of instantaneous and average rates of change (gradients of tangents and chords) in numerical, algebraic and graphical contexts}
- Calculate or estimate the area under a graph and interpret the result as a distance

Knowledge

- Know and understand what a graph is and how it can represent a relationship
- Move freely between different numerical, algebraic, graphical and diagrammatic representations
- Recognise different forms of a linear equation, including slope-intercept form y = mx + c
- Understand that every point on a graph represents the solution to an equation, and that the intersections of two graphs represent solutions that satisfy both relationships
- Recognise the different types of non-linear graphs at GCSE level (quadratic, cubic, reciprocal {exponential}
- Understand the relationship between solutions to an equation and roots of a graph
- {recognise and use the equation of a circle with centre at the origin}
- {Know that the gradient of a point on a line or curve is the instantaneous rate of change at that point in the function}

Rationale

This module builds on earlier study of straight line graphs in years 9 and 10. Students plot straight lines from a given equation, and find and interpret the equation of a straight line from a variety of situations and given information. There is opportunity to revisit graphical solutions of simultaneous equations. Higher tier students also study the equations of perpendicular lines.

Then, students develop their knowledge of non-linear graphs, looking at quadratic, cubic and reciprocal graphs. They will be able to recognise the different shapes. They find the roots of quadratics graphically, and will revisit this in addition to turning points when they look at algebraic methods in the next unit. Higher tier students also look at simple exponential graphs and the equation of a circle.v The equation of the tangent to a circle is covered later when the circle theorem of tangent/radius is met. Higher students also extend their understanding of gradient to include the instantaneous rates of change.

Lastly, learners revise conversion graphs and reflection in straight lines. Students study other real-life graphs, including speed/distance/time, constructing and interpreting these. Higher tier students also investigate the area under a curve.

Autumn (Teacher B): Algebra

Skills

- Use algebra to support and construct arguments {and proofs}
- Solve linear equations and inequalities in one variable
- Simplify and manipulate quadratic expressions by factorising, including the difference of two squares {and non-monic quadratics of the form ax^2 + bx + c}
- Solve quadratic equations {including those that require rearrangement} algebraically by factorising {by completing the square and using the quadratic formula{
- Identify and interpret roots; deduce roots algebraically {and turning points by completing the square}
- {solve two simultaneous equations in one variable, where one equation is quadratic}
- Find approximate solutions using a graph
- Translate simple situations or procedures into algebraic expressions or formulae
- Derive an equation (or two simultaneous equations), solve the equation(s) and interpret the solution
- {find approximate solutions to equations numerically using iteration}
- Represent the solutions to equations and inequalities on a number line and using set notation
- Apply Pythagoras' theorem and the trigonometric ratios to find angles and lengths in right angled triangles {and where possible in general triangles} in 2D {and 3D}.

Knowledge

- Know the difference between an equation and an identity; argue mathematically to show algebraic expressions are equivalent
- Recognise quadratic expressions and use correct mathematical notation
- Interpret simple expressions as functions with inputs and outputs; {interpret the reverse process as the 'inverse function'; interpret the succession of two functions as a 'composite function'}
- Use function notation to express the value of a function {the inverse function and composite functions}
- Relate ordinary algebraic functions to the trigonometric functions

Rationale

Learners begin the Autumn term by expanding and factorising with a single bracket before moving on to quadratic expressions. The use of algebra tiles to develop conceptual understanding is encouraged throughout. Context questions are included to revisit e.g. area and Pythagoras' theorem.

Following this, learners consolidate and build on their study of changing the subject in Year 9. The unit begins with a review of solving equations and inequalities before moving on to rearrangement of both familiar and unfamiliar formulae. Checking by substitution is encouraged throughout. Higher students also study solving equations by iteration.

Lastly, learners study functions. As well as introducing formal function notation, this unit brings together and builds on the work on quadratic functions and graphs undertaken with Teacher A. This is also an opportunity to revisit trigonometric functions.

Spring (Teacher A): Reasoning	
Skills	• {construct and} interpret equations that describe direct and inverse proportion • Reason deductively in geometry, number and algebra, including using geometrical constructions • {apply and prove the standard circle theorems concerning angles, radii, tangents and chords, and use them to prove related results} • Interpret and use bearings • Apply addition and subtraction of vectors, multiplication of vectors by a scalar, and diagrammatic and column representations of vectors; {use vectors to construct geometric arguments and proofs} • Make and test conjectures about the generalisations that underlie patterns and relationships; look for proofs or counter-examples; begin to use algebra to support and construct arguments {and proofs} • Deduce expressions to calculate the n th term of linear {and quadratic} sequences • Solve two simultaneous equations in two variables (linear/linear {or linear/quadratic}) algebraically; find approximate solutions using a graph
Knowledge	• Understand how to communicate mathematically, how deductive reasoning works in different contexts • Compare lengths, areas and volumes using ratio notation and/or scale factors; make links to similarity • Understand that X is inversely proportional to Y is equivalent to X is proportional to $1Y$ • Extend and formalise their knowledge of ratio and proportion, including trigonometric ratios, in working with measures and geometry, and in working with proportional relations algebraically and graphically
Rationale	Students develop their multiplicative reasoning in a variety of contexts, from simple scale factors through to complex equations involving direct and inverse proportion. They link inverse proportion with the formulae for pressure and density. There is also the opportunity to review ratio problems. Students consolidate their knowledge of angles facts and develop increasingly complex chains of reasoning to solve geometric problems. Higher tier students revise the first four circle theorems studied in Year 10 and learn the remaining theorems. Students also revisit vectors and the key topics of Pythagoras' theorem and trigonometry. Students develop their algebraic reasoning by looking at more complex situations. They use their knowledge of sequences and rules to made inferences, and Higher tier students move towards formal algebraic proof. Forming and solving complex equations, including simultaneous equations, is revisited. Higher tier students also look at solving inequalities in more than one variable.

Spring (Teacher B): Language and communication

Skills

- Describe translations as 2D vectors
- Reason deductively in geometry, number and algebra, including using geometrical constructions
- Interpret and use fractional {and negative} scale factors for enlargements
- {describe the changes and invariance achieved by combinations of rotations, reflections and translations}
- Recognise, sketch and interpret graphs of {the trigonometric functions (with arguments in degrees) for angles of any size}
- {sketch translations and reflections of the graph of a given function}
- Calculate the probability of independent and dependent combined events, including using tree diagrams and other representations, and know the underlying assumptions
- {calculate and interpret conditional probabilities through representation using expected frequencies with two-way tables, tree diagrams and Venn diagrams}

Knowledge

- Explore what can and cannot be inferred in statistical and probabilistic settings, and express their arguments formally
- Know the difference between an equation and an identity; argue mathematically to show algebraic expressions are equivalent, and use algebra to support and construct arguments {and proofs}
- Apply the concepts of congruence and similarity
- Apply systematic listing strategies, {including use of the product rule for counting}
- Construct and interpret plans and elevations of 3D shapes
- Make and use connections between different parts of mathematics to solve problems
- {change recurring decimals into their corresponding fractions and vice versa}
- Apply addition and subtraction of vectors, multiplication of vectors by a scalar, and diagrammatic and column representations of vectors; {use vectors to construct geometric arguments and proofs}

Rationale

Students revise and extend their learning from KS3, exploring all the transformations and constructions, relating these to symmetry and properties of shapes when appropriate. There is an emphasis on describing as well as performing transformations as using the language promotes deeper thinking and understanding. Higher tier students extend their learning to explore the idea of invariance and look at trigonometric graphs as a vehicle for exploring graph transformations.

This block is another vehicle for revision as the examinations draw closer. Students look at organisation information, with Higher tier students extending this to include the product rule for counting. Links are made to probability and other aspects of Data Handling such as describing and comparing distributions and scatter diagrams. Plans and elevations are also revised. You can adapt the exact content to suit the needs of your class.

This is another block designed to be adapted to suit the needs of your class. Examples of communication in various areas of mathematics are provided in order to highlight gaps in knowledge that need addressing in the run up to the examinations. "Show that" is used to encourage students to communicate in a clear mathematical fashion, and this skill should be transferred to their writing of solutions to any type of question.