

Autumn (Teacher A): Similarity

Skills

- Compare lengths, areas and volumes using ratio notation and/or scale factors;
- Interpret and use fractional {and negative} scale factors for enlargements
- Apply the concepts of congruence and similarity, including the relationships between lengths, {areas and volumes} in similar figures
- Use mathematical language and properties precisely
- Make and test conjectures about the generalisations that underlie patterns and relationships; look for proofs or counter-examples
- Develop knowledge, in part through solving problems and evaluating the outcomes, including multi-step problems
- Apply Pythagoras' Theorem and trigonometric ratios to find angles and lengths in right-angled triangles {and, where possible, general triangles} in two {and three} dimensional figures
- Select appropriate concepts, methods and techniques to apply to unfamiliar and non-routine problems

Knowledge

- Concepts of congruence, similarity and invariance
- Make links between ratio and similarity
- Extend and formalise their knowledge of ratio and proportion in working with measures and geometry
- Extend and formalise their knowledge of ratio and proportion, including trigonometric ratios
- Pythagoras' theorem and when it can be applied successfully
- Know the exact values of $\sin \theta$, $\cos \theta$, $\tan \theta$ for required angles
- {know and apply the sine rule and cosine rule to find unknown lengths and angles}
- {know and apply to calculate the area, sides or angles of any triangle}
- Modelling and how models can be improved by comparing to real life outcomes
- Make and use connections between different parts of mathematics to solve problems

Rationale

Building on their experience of enlargement and similarity in previous years, this unit extends students' experiences and looks more formally at dealing with topics such as similar triangles. ICT is used to demonstrate what changes and what stays the same when manipulating similar shapes. Parallel line angle rules are revisited to support establishment of similarity. Congruency is reintroduced through considering what information is needed to produce a unique triangle. Higher level content extends enlargement to explore negative scale factors, and also looks at establishing that a pair of triangles are congruent through formal proof.

Trigonometry is then reintroduced as a special case of similarity within right-angled triangles. Emphasis is placed throughout the steps on linking the functions to ratios, rather than just functions. This key topic is introduced early in Year 10 to allow regular revisiting e.g. when looking at bearings. For the Higher tier, calculation with trigonometry is covered now and graphical representation is covered in Year 11

Autumn (Teacher B): Developing algebra

Skills	<ul style="list-style-type: none"> • Translate simple situations or procedures into algebraic expressions or formulae; derive an equation (or two simultaneous equations), solve the equation and interpret the solution • Select appropriate concepts, methods and techniques to apply to unfamiliar and non-routine problems; interpret their solution in the context of the given problem. • Recognise, sketch and interpret graphs of linear functions and quadratic functions. • Solve linear inequalities in one {or two} variable{s}, {and quadratic inequalities in one variable}; represent the solution set on a number line, {using set notation and on a graph} • Factorising quadratic expressions of the form $x^2 + bx + c$; solve quadratic equations algebraically by factorising (Higher only at this stage) • Solve two simultaneous equations in two variables (linear/linear {or linear/quadratic}) algebraically;
Knowledge	<ul style="list-style-type: none"> • Consolidate their algebraic capability from key stage 3 and extend their understanding of algebraic simplification and manipulation to include quadratic expressions • Features of linear and quadratic expressions • Understanding of key symbols and concepts in set theory, and how they can be used to represent sets of numbers inexplicitly • Model situations mathematically and express the results using a range of formal mathematical representations, reflecting on how their solutions may have been affected by any modelling assumptions
Rationale	<p>Students will have covered both equations and inequalities at KS3, and this unit offers the opportunity to revisit and reinforce standard techniques and deepen their understanding. Looking at the difference between equations and inequalities, students will establish the difference between a solution and a solution set; they will also explore how number lines and graphs can be used to represent the solutions to inequalities. As well as solving equations, emphasis needs to be placed on forming equations from given information. This provides an excellent opportunity to revisit other topics in the curriculum such as angles on a straight line/in shapes/parallel lines, probability, area and perimeter etc. Factorising quadratics to solve equations is covered in the Higher strand here and is revisited in the Core strand in Year 11</p> <p>Students then move on to the solution of simultaneous equations by both algebraic and graphical methods. The method of substitution will be dealt with before elimination, considering the substitution of a known value and then an expression. With elimination, all types of equations will be considered, covering simple addition and subtraction up to complex pairs where both equations need adjustment. Links will be made to graphs and forming the equations will be explored as well as solving them. The Higher strand will include the solution of a pair of simultaneous equations where one is a quadratic, again dealing with factorisation only at this stage.</p>

Spring (Teacher A): Geometry

Skills

- Interpret and use bearings
- Compare lengths using scale factors
- Apply Pythagoras' Theorem and trigonometric ratios to find angles and lengths in right-angled triangles {and, where possible, general triangles} in two dimensional figures
- {know and apply the sine rule and cosine rule to find unknown lengths and angles}
- Use mathematical language and properties precisely
- Calculate arc lengths, angles and areas of sectors of circles
- Calculate surface areas and volumes of spheres, pyramids, cones and composite solids
- Apply and prove the standard circle theorems concerning angles, radii, tangents and chords, and use them to prove related results
- Apply addition and subtraction of vectors, multiplication of vectors by a scalar, and diagrammatic and column representations of vectors; {use vectors to construct geometric arguments and proofs}.
- Describe translations as 2D vectors

Knowledge

- Understand rules for using bearings
- Identify and apply circle definitions and properties, including: centre, radius, chord, diameter, circumference, tangent, arc, sector and segment
- Know and recognise the standard circle theorems
- Understand the concept of a vector, and how it is different to other mathematical entities such as variables, constants and functions.
- Reason deductively in geometry, number and algebra, including using constructions
- Make and use connections between different parts of mathematics to solve problems

Rationale

As well as the formal introduction of bearings, this block provides a great opportunity to revisit other materials and make links across the mathematics curriculum. Accurate drawing and use of scales will be vital, as is the use of parallel line angles rules; all of these have been covered at KS3. Students will also reinforce their understanding of trigonometry and Pythagoras from earlier this year, applying their skills in another context as well as using mathematics to model real-life situations.

The formulae for arc length and sector area are built up from students' understanding of fractions They are also introduced to the formulae for surface area and volume of spheres and cones; here higher students can enhance their knowledge and skills of working with area and volume ratios. Higher tier students are also introduced to four of the circle theorems; the remaining theorems will be introduced in Year 11 when these four will be Revisited.

Students will have met vectors to describe translations during KS3 This will be revisited and used as the basis for looking more formally at vectors, discovering the meaning of $-a$ compared to a to make sense of operations such as addition, subtraction and multiplication of vectors. This will connect to exploring 'journeys' within shapes linking the notation AB with $b - a$ etc. Higher tier students will then use this understanding as the basis for developing geometric proof, making links to their knowledge of properties of shape and parallel lines.

Spring (Teacher B): Proportions and proportional change

Skills

- Use ratio notation, including reduction to simplest form
- Divide a given quantity into two parts in a given *part : part* or *part : whole* ratio; express the division of a quantity into two parts as a ratio.
- Use compound units such as speed, unit pricing and density to solve problems.
- Compare lengths, areas and volumes using ratio notation and/or scale factors
- Interpret percentages multiplicatively, express one quantity as a percentage of another, compare two quantities using percentages,
- Solve problems involving percentage change, including: percentage increase, decrease and original value problems and simple interest in financial mathematics.
- Set up, solve and interpret the answers in growth and decay problems, including compound interest {and work with general iterative processes}.
- Calculate the probability of independent and dependent combined events, including using tree diagrams and other representations, and know the underlying assumptions.
- {Calculate and interpret conditional probabilities through representation using expected frequencies with two-way tables, tree diagrams and Venn diagrams}.

Knowledge

- Know and use ratio notation as a multiplicative comparison
- Relate the language of ratios and the associated calculations to the arithmetic of fractions and to linear functions.
- Understand the relationship between ratios and rates
- Link ratio and similarity in geometry
- Apply the concepts of congruence and similarity, including the relationships between lengths, {areas and volumes} in similar figures.
- Define a percentage as “per 100” and work with percentages greater than 100%
- Awareness of how percentages are used in real life, such as VAT and interest
- Know that probabilities take values between 0 and 1, and that the probabilities of an exhaustive set of mutually exclusive events sum to 1.
- Use a probability model to predict the outcomes of future experiments; understand that empirical unbiased samples tend towards theoretical probability distributions, with increasing sample size.

Rationale

This block builds on KS3 work on ratio and fractions, highlighting similarities and differences and links to other areas of mathematics including both algebra and geometry. The focus is on reasoning and understanding notation to support the solution of increasingly complex problems that include information presented in a variety of forms. The bar model is a key tool used to support representing and solving these problems.

Although percentages are not specifically mentioned in the KS4 national curriculum, they feature heavily in GCSE papers and this block builds on the understanding gained in KS3. Calculator methods are encouraged throughout and are essential for repeated percentage change/growth and decay problems. Use of financial contexts is central to this block, helping students to maintain familiarity with the vocabulary they are unlikely to use outside school.

This block also builds on KS3 and provides a good context in which to revisit fraction arithmetic and conversion between fractions, decimals and percentages. Tables and Venn diagrams are revisited and understanding and use of tree diagrams is developed at both tiers, with conditional probability being a key focus for Higher tier students.

Summer (Teacher A): Delving into data and expressions

Skills

- Use describe, interpret and compare observed distributions of a single variable through appropriate graphical representation involving discrete, continuous and grouped data
- Construct and interpret appropriate tables, charts, and diagrams, including frequency tables, bar charts, pie charts, and pictograms for categorical data, and vertical line (or bar) charts for ungrouped and grouped numerical data
- Describe, interpret and compare observed distributions of a single variable through: appropriate graphical representation involving discrete, continuous and grouped data; and appropriate measures of central tendency (mean, mode, median) and spread (range, consideration of outliers)
- Interpret and construct tables and line graphs for time series data {construct and interpret diagrams for grouped discrete data and continuous data, i.e. histograms with equal and unequal class intervals and cumulative frequency graphs}
- Interpret, analyse and compare the distributions of data sets from univariate empirical distributions through appropriate graphical representation involving discrete, continuous and grouped data, {including box plots}
- Apply statistics to describe a population
- Interpret, analyse and compare the distributions of data sets from univariate empirical distributions through appropriate measures of central tendency (including modal class) and spread {including quartiles and inter-quartile range}
- Simplify and manipulate algebraic expressions (including those involving surds {and algebraic fractions} by factorising quadratic expressions of the form $x^2 + bx + c$

Knowledge

- Know the different types of data that exist, and the ways they may be represented
- Knowledge of the different ways of expressing central tendency and spread, and the limitations of these statistics
- Infer properties of populations or distributions from a sample, whilst knowing the limitations of sampling
- Understand the differences between charts suitable for discrete and continuous numerical data
- Know the difference between an equation and an identity
- Argue mathematically to show algebraic expressions are equivalent, and use algebra to support and construct arguments {and proofs}

Rationale

This block builds on KS3 work on the collection, representation and use of summary statistics to describe data. Much of the content is familiar, both from previous study within and beyond mathematics (including Geography and Science) and from everyday life. The steps have been chosen to balance consolidation of existing knowledge with extending and deepening, particularly in terms of interpretation of results and evaluating and criticising statistical methods and diagrams. For students following Higher tier, there is additional content relating to continuous data including histograms, cumulative frequency diagrams, box plots and associated measures such as quartiles and the interquartile range. Again the emphasis with these topics should be on interpretation (particularly in making comparisons) and not just construction.

The second part of this module builds on the Autumn term learning of equations and inequalities, providing revision and reinforcement for Foundation tier students and an introduction to algebraic fractions for those following the Higher tier. This also allows all students to revise fraction arithmetic to keep their skills sharp. Algebraic argument and proof are considered, starting with identities and moving on to consider generalised number.

Summer (Teacher B): Using number

Skills

- Calculate exactly with fractions, {surds} and multiples of π ; {simplify surd expressions involving squares and rationalise denominators}
- {change recurring decimals into their corresponding fractions and vice versa}
- Describe and continue sequences
- Deduce expressions to calculate the n th term of linear {and quadratic} sequence
- Recognise and use sequences of square and cube numbers
- {estimate powers and roots of any given positive number}
- Calculate with roots, and with integer {and fractional} indices
- Calculate with numbers in standard form $A \times 10^n$, where $1 \leq A < 10$ and n is an integer
- Simplifying expressions involving sums, products and powers, including the laws of indices

Knowledge

- Factors, multiples, primes, HCF and LCM
- Apply and interpret limits of accuracy when rounding or truncating, {including upper and lower bounds}
- Recognise and use sequences of triangular, simple arithmetic progressions, Fibonacci type sequences, quadratic sequences, and simple geometric progressions (rn where n is an integer, and r is a positive rational number {or a surd}) {and other sequences}
- Develop their use of formal mathematical knowledge to interpret and solve problems, including in financial contexts
- Make and use connections between different parts of mathematics to solve problems
- Know the rules for expressions written in standard form, and the uses of standard form in contexts such as science and engineering

Rationale

This block revises and builds on KS3 content for calculation. Mental methods and using number sense are to be encouraged alongside the formal methods for all four operations with integers, decimals and fractions. Where possible this should be covered through problems, particularly multi-step problems in preparation for GCSE. The limits of accuracy of truncation are explored and compared to rounding, and Higher tier students will look at all aspects of irrational numbers including surds.

Learners then review prime factorisation and associated number content such as HCF and LCM. Sequences is extended for Higher tier to include surds and finding the formula for a quadratic sequence.

Finally, learners consolidate the previous two blocks focusing on understanding powers generally, and in particular in standard form. Negative and fractional indices are explored in detail. Again, much of this content will be familiar from KS3, particularly for Higher tier students, so this consolidation material may be covered in less than two weeks allowing more time for general non-calculator and problem-solving practice. To consolidate the index laws, these can be revisited in the next block when simplifying algebraic expressions.